The Tenability of Desire-as-Belief[*]

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Draft

1 Introduction

In decision theory, preferences of agents are represented by an expected value function V, in the following technical sense: an agent prefers proposition A to B if and only if $V(A) > V(B)$. V and the probability function P which represents the agent's credence, or degrees of belief, satisfy an additivity constraint. For incompatible A and B to which P assigns positive probability, the following equation holds:

1) $V(A \vee B) = \frac{P(A)V(A)+P(B)V(B)}{P(A)+P(B)}$

We may think of the expected value function V as determined by an "ultimate value function" V_U, which is only defined on point-propositions, and a probability function P. Since all propositions are equivalent to disjunctions of point-propositions, equation 1) fully defines the expected value function, or "desire function," as we may also call it.[1]

What represents the agent is the package of the two functions P and V_U. We may wonder whether there is an alternative representation by the credence function P alone. For example, we can ask whether an agent prefers A to B if and only if $P(A^\circ) > P(B^\circ)$, for some function $^\circ$. Heuristically, we could think of A° as the proposition that A is desirable, or (subjectively)

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[1]Qualification: This is true only if there no more than finitely many point-propositions. Otherwise, 1) needs to be replaced by an equation with countable sums or integrals.

valuable. The thesis that agents can be thus represented have come to be known as the "Desire-as-Belief" thesis. It will turn out that we need to distinguish various versions of it. Nonetheless, it is convenient to use the term 'DAB' to refer to the generic, not fully precise thesis.

Offhand, DAB appears as a thesis that is true of some possible agents but not of others. Plausibly, some agents have "non-intellectualized" desires, which do not find a reflection in some belief about the value of the object of the desire. They may lack the conceptual resources to have an attitude to A°, or they may lack the belief even though they would have all the pertinent concepts. On the other hand, it is hard to see why it should not be possible for an agent to have their desires in A match their beliefs in A°. In Lewis [1988], where DAB was first discussed, David Lewis offers a fictional candidate for satisfying DAB: Frederic, the slave of duty from Gilbert and Sullivan's "The Pirates of Penzance." It might turn out that such a character is impossible, but it would be surprising. Presumably, psychology, broadly construed to include some parts of philosophy of mind, needs to tell us whether it is true for some agents, or perhaps for all human agents, that there is a function $^\circ$ such that agents believe A° more strongly than B° whenever their actions reveal a preference of A to B.

Lewis [1988] argued that this appearance of contingency is illusory: DAB cannot be true of any agent, and we do not need to engage in empirical psychology to establish that. According to Lewis, DAB clashes with non-negotiable principles of decision theory. Several subsequent papers have discussed Lewis's claim, but none has attempted to fully rehabilitate DAB.[2]

Here, I argue that DAB is formally tenable. I do not defend its truth, but merely want to vindicate the initially plausible thought that it is a broadly empirical question whether DAB is true, which is not to be settled by mathematical results alone.

Lewis introduces DAB as articulating a distinctly anti-Humean conception about the relationship between reason and the passions. However, it is not obvious that DAB needs to be interpreted as anti-Humean. Further-

[2]The case against DAB was furthered in Lewis [1996] and Arló Costa et al. [1995]. Price [1989] and Hájek and Pettit [2004] defend theses related to DAB, more on which below.

more, it is obvious, and acknowledged by Lewis, that DAB is not the only implementation of an anti-Humean agenda. In this note, questions about the relationship between DAB and anti-Humeanism will be set aside. I likewise do not discuss the question whether DAB would yield us "objective ethics," as Lewis seems to suppose.

The plan of this note is as follows. In section 2, I report Lewis's argument against DAB. In section 3, I outline a strategy how to rehabilitate DAB. Sections 4 and 5 implement that strategy.

2 Lewis's Argument against DAB

Lewis argues that DAB is untenable because it collides with decision theory:

> Decision Theory is an intuitively convincing and well worked-out formal theory of belief, desire, and what it means to serve our desires according to our beliefs. It is of course idealized, but surely it is fundamentally right. If an anti-Humean Desire-as-Belief Thesis collides with Decision Theory, it is the Desire-as-Belief Thesis that must go. [Lewis, 1988, p. 45]

I will not present Lewis's formal argument here. However, it is important to be clear about what his result is.

Often, DAB is stated simply as an equation:

2) $V(A) = P(A^\circ)$.

As emphasized by Hájek and Pettit [2004], that equation is not a claim, since no particular V, P, A, and A° are given. Different ways of prefixing 2) with quantifiers result in very different theses. To state the claim that Lewis refutes to a first approximation, let I be some input, typically given by experience, and let P_I and V_I be the results of applying Jeffrey conditionalization on I to P and V, respectively. Lewis shows that trivial cases aside, there is no pair $\langle P, V\rangle$ that satisfies the following condition:

3) $\forall A \exists A^\circ \forall I\ V_I(A) = P_I(A^\circ)$.

In fact, Lewis's result is stronger. To appreciate it fully, we need to look more closely at the operation of Jeffrey conditionalization. In Lewis's presentation, the input to Jeffrey conditionalization consists of an *originating partition*, a *distribution*, and an *amount*. The originating partition is an n-tuple of propositions that are pairwise inconsistent and whose disjunction is a tautology. The distribution is an n-tuple of real numbers that sum to zero. The amount is a real number.[3] I will use the variable I to range over inputs.[4]

Jeffrey conditionalizing a probability function P on input I results in a probability function P_I defined by:

4) $P_I(A) = P(A)[1 + px]$, where $p = \sum_i \frac{P(A|E_i)d_i}{P(A)}$.

Further, we need to define what the result of applying Jeffrey conditionalization to an expected value function rather than a probability function is. Recall that an expected value function V is determined by an ultimate value function V_U and P, via equation 1). To obtain V_I, we take the same V_U, but spread it over disjunction by using P_I instead of P.[5] V_I is then expressed as a function of V as follows:

5) $V_I(A) = V(A)\frac{[1+rx]}{[1+px]}$, where $r = \sum_i \frac{V(AE_i)P(A|E_i)d_i}{V(A)P(A)}$.

Lewis points out that the originating partition and the distribution ought not always to determine the amount. He assumes that if I is an input with originating partition p, distribution d and amount x, there is another non-zero amount y such that p, d and y is an input satisfying all constraints as well. Let $\Phi(P, V)$ be the class of inputs I to $\langle P, V\rangle$ and A that change both $P(A)$ and $V(A)$. Given the above assumption, Lewis does not just refute 2), but also the weaker claim we get by replacing '$\forall I$' by '$\exists\ I \in \Phi(P, V)$'.

Hájek and Pettit [2004] suggest to get around this result by flipping the quantifiers in 3): A° may depend on the input I.[6] They formulate a thesis they call "Indexical Desire-as-Belief." That thesis is rather weak, but they

[3]In the more familiar special case of conditionalization of P on A, the originating partition is $\langle A, \neg A\rangle$, the distribution $\langle 1, -1\rangle$, and the amount $1 - P(A)$.

[4]The same input can be represented by multiplying the distribution with some number and dividing the amount by the same number.

[5]In other words, we make what Lewis calls the "Invariance Assumption."

[6]In their presentation, on the pair $\langle P, V\rangle$.

point out that it may be supplemented by further, more substantial conditions. Here, I want to show that 3) can be weakened in such a way that it still a relatively strong claim, but immune to any triviality results.

3 A Tenable Version of Desire-as-Belief

Rather than reversing the order of quantifiers, I propose to restrict the universal quantifiers that bind the proposition A and the input I. Moreover, I argue that the updating operation is not quite Jeffrey conditionalization, but what I call *contracted Jeffrey conditionalization*.

On my proposal, A° may be read as "If A, then things are good." If we let G stand for the proposition that things are good, or as desired, and $\rightarrow$ for the indicative conditional, then we can also write A° as $A \rightarrow G$. However, this interpretation is not compulsory; I do not intend to make claims about conditionals in this note.

To construct a function $^\circ$ that vindicates Desire as Belief, I proceed in two steps. First, I show that a related thesis, called "Desire by Necessity" or "DBN", is formally tenable. A pair $\langle P, V \rangle$ satisfies DBN if there is a proposition G such that for all A, $V(A) = P(G|A)$. Lewis himself acknowledges that his argument leaves DBN unscathed. To show positively that DBN is tenable, I present models for it in the next section.

In the second step, I use the tenability of DBN to argue for the tenability of DAB. For this purpose, I consider models in which there is, for every A, a proposition A° such that $P(A^\circ) = P(G|A)$. Putting the two steps together, we obtain $V(A) = P(A^\circ)$.

The last paragraph will have set off alarm bells in some readers. For the well-known triviality results for the hypothesis of the Conditional Construal of Conditional Probability, or 'CCCP' for short, seem to show that there cannot be such a proposition A° for every A. Indeed, Lewis himself cites the triviality results for CCCP as an obstacle for DAB. I will attempt to silence these alarm bells in section 5. To anticipate: I will make use of one of the partial positive tenability results about CCCP, due to van Fraassen [1976], and argue that it covers what the defender of DAB needs.

Before explaining the details in the next two sections, I want to indicate briefly how my proposal avoids Lewis's result. As noted above, the basic moves are restricting the universal quantifiers, and modifying Jeffrey conditionalization. Once we have suitable restrictions under which DAB is tenable, there remains the question whether these restrictions can be motivated philosophically. I want to argue that there does not need to be a proposition A° satisfying the equation for every A. Desire as Belief is proposed as an additional constraint on decision theory. I mentioned that in decision theory, agents' preferences are represented by the function V. However, it is not essential to decision theory that the preference relation is defined for all propositions on which the credence function P is defined. It might be a partial relation. If we want to apply decision theory to decision problems, we need only make sure that the preference relation is defined for propositions that describe actions that an agent can perform. Clearly, not all propositions are of the kind that an agent has the power to make true, and thus candidates for consideration in a decision problem. For example, the proposition that the sun rises tomorrow is not, for typical mortal agents. More pertinently, the proposition that A is good, or desirable, does not describe an action, even if A does.

Moreover, not every proposition is of the sort that can serve as input to updating rules; or in the context of Jeffrey conditionalization, such that it can be a member of an originating partition. Again, the proposition that A is good, or desirable, is arguably not a candidate.

More needs to be said about the sense in which DAB turns out to be formally tenable. In one sense, being formally tenable is nothing more than avoiding triviality. The Valuation Lemma in the next section shows that DBN ("Desire by Necessity") is tenable in this weak sense. But a proposed constraint on credence and expected value functions may not lead to triviality, but still have very substantive implications for the preference relations of agents that obey the constraint. I want to argue that DAB is not of that kind. Any pattern of preferences among acts can be represented by a pair $\langle P, V \rangle$ for which DAB holds, and continues to hold under updating.

I am certainly not claiming that every agent needs to be thus represented. But it seems to me that one and the same preference relation may be represented by different pairs of functions. To be sure, preferences among acts constrain what beliefs and desires an agent can have. But unless we subscribe to a version of behaviourism or verificationism, we need not suppose that such preferences fully determine the beliefs and desires.

Representation theorems typically have an existence and a uniqueness part. The uniqueness part says that all representing functions can be transformed into each other, and specifies the pertinent class of transformations. But this result depends on the assumption that preferences are defined for all propositions that are in the domain of the agent's credence function. If what I suggested in the last paragraph is right, this assumption is unjustified. In applying decision theory, we need only a representation of the agent of the following kind: for any A and B *that could occur in a decision problem*, the agent prefers A to B if and only if $V(A) > V(B)$. In the rest of this paper, I want to show that for any agent, there is a representation in this weaker sense for which DAB holds.

4 Models for Desire by Necessity

As the first step of my defence of the tenability of DAB, I show that there are models of the thesis Lewis calls "Desire by Necessity," or "DBN." While 2) is the characteristic equation of DAB, 6) is the characteristic equation of DBN:

6) $V(A) = P(G|A)$

DBN is a special case of yet another thesis in the neighborhood, Desire as Conditional Belief or "DACB," with the equation $V(A) = P(A°|A)$. We obtain DBN by replacing $A°$ by G in DACB.[7]

Lewis suggested that his triviality arguments did not indict DBN. However, to show positively that DBN is formally tenable, and thus will not fall victim to future triviality arguments, we need to show that there are models in which it holds.

I first introduce some standard terminology and machinery from probability theory. Let W be a set of point-propositions, or possible worlds, and $\mathcal{F}$

[7] While DACB has been advocated by Huw Price [1989], DBN has no explicit defenders in print, though it is discussed in Lewis [1996].

an algebra of subsets of W, typically called "events" by probability theorists or "propositions" by philosophers. A probability function P is a normalized, non-negative and finitely additive function on $\mathcal{F}$. A probability function is regular if it assigns zero only to the null element of $\mathcal{F}$. In other words, a regular probability function assigns positive numbers to all point-propositions. In the following, I will only quantify over regular probability functions. Jeffrey's decision theory requires agents to have regular credence functions, reflecting the fact that we cannot ever be certain of any non-tautologous proposition. Likewise, when I quantify over inputs I to Jeffrey conditionalization, I restrict myself to inputs that lead to regular probability functions.[8]

It is familiar that given a conditional probability function, i.e. a two-place function P, fixing the second argument place produces a one-place probability function. If $P(A) > 0$, then P' defined by $P'(B) = P(B|A)$ for all $B \in F$ is the *conditionalization* of P on A.[9]

What happens if we fix the first argument place of P? Q defined by $Q(B) = P(A|B)$ for all $B \in \mathcal{F}$ such that $P(B) \neq 0$ is not a probability function, since it is not additive: $Q(B) + Q(\neg B) = P(A|B) + P(A|\neg B) = \frac{P(A \wedge B)}{P(B)} + \frac{P(A \wedge \neg B)}{P(\neg B)} > P(A \wedge B) + P(A \wedge \neg B) = P(A) = P(A|T) = Q(T)$. But Q thus defined is an expected value function, as asserted by the Valuation Lemma below. For lack of a better term, I call Q thus defined a *valuation* of P on A, and denote it by P_A.[10] In the literature, this notation is used for the conditionalization of P on A, but there will be no confusion here because I use variables I, I', etc. for the input of Jeffrey conditionalization.

We have now introduced two operations, Jeffrey conditionalization and valuation. We need to ask whether the order in which we apply these to probability functions makes a difference. This question is answered by the Valuation Lemma.

[8]Given the restriction to regular functions, conditionalization is no longer a special case of Jeffrey conditionalization.

[9]Perhaps it accords better with standard usage to say that P' is the conditionalization on A not of P, but of the one-place probability function P^* such that $P^*(B) = P(B|T)$. I will sometimes use the same letter P with harmless ambiguity for both a probability function and a conditional probability function related to P by the ratio formula.

[10]It is clear that given the assumption that $P(B|T) = 0$ only if $B = \perp$, then if a function is a valuation of P on both A and A', then $A = A'$.

Valuation Lemma If P is a probability function on $\mathcal{F}$, $G \in \mathcal{F} \setminus \bot$, and I an input, then P_{IG} is an expected value function (relative to P_I) and equals P_{GI}.

Proof. First, I show that P_{IG} is an expected value function relative to P_I. If A and B are compatible, then $P_{IG}(A \vee B) = P_I(G|A \vee B) = \frac{P_I(G\wedge(A\vee B))}{P_I(A\vee B)} = \frac{P_I(G\wedge A)+P_I(G\wedge B))}{P_I(A)+P_I(B)} = \frac{P_I(A)P_I(G|A)+P_I(B)P_I(G|B)}{P_I(A)+P_I(B)} = \frac{P_I(A)P_{IG}(A)+P_I(B)P_{IG}(B)}{P_I(A)+P_I(B)}$, and hence P_{IG} is an expected value function relative to P_I.

Second, it needs to be shown that for all A, $P_{IG}(A)$ equals $P_{GI}(A)$. The former is by definition $P_I(G|A)$, which is then unpacked using the ratio formula and equation 4). The latter is unpacked using equation 5), with P_G in the place of V. It is straightforward, although tedious, to verify that they are equal.

For the purpose of showing that DBN is tenable, it is crucial that $P_{IG} = P_{GI}$, that "the diagram commutes." Lewis rejects DAB because even if $V(A)$ and $P(A^\circ)$ coincide, they will be shorn apart by Jeffrey conditionalization. The Valuation Lemma shows us that DBN is not vulnerable to that objection: DBN continues to hold under Jeffrey conditionalization.

The Valuation Lemma already shows us that DBN is not susceptible to triviality results. It does not show us, however, that any given agent, whatever her preferences, can be represented with a pair $\langle P, V\rangle$ for which DBN holds. This will be shown by the DBN Extension Lemma below.

The idea is as follows: Suppose the preferences of an agent are represented by a pair $\langle P, V\rangle$ of functions defined over an algebra $\mathcal{F}$ of subsets of W. Then we construct a new set of point-propositions W', and an algebra $\mathcal{F}'$ of its subsets, in which there is a proposition G such that P is the valuation of P on G. Or better, such that a function associated with P is the valuation, on G, of a function associated with V.

The members of W', the "new worlds," are ordered pairs, whose first member is an "old world," i.e. a member of W, and whose second member is either The Good or The Bad. Every old world thus splits into two, a good one and a bad one.[11] G is then the proposition that consists of all the good

[11]If we think that goodness metaphysically supervenes on non-evaluative features of a world, then we should not think of the members of W' as metaphysically possible worlds.

worlds and none of the bad ones. If a proposition $A \in \mathcal{F}'$ contains exactly those good worlds in $B \in \mathcal{F}$, and exactly those bad worlds in $C \in \mathcal{F}$, then I will denote it by $\langle B, C \rangle$.

Let W' be the smallest set that contains $\langle w, Good \rangle$ and $\langle w, Bad \rangle$ whenever $w \in W$. The algebra of subsets of W' generated by $G = \{\langle w, Good \rangle : w \in W\}$ and $\{\langle w, Good \rangle : w \in A\} \cup \{\langle w, Bad \rangle : w \in A\}$, for every $A, B \in \mathcal{F}$ is the *G-algebra* of W.

The next lemma needs some terminology. A function $'$ is an *embedding* of an algebra $\mathcal{F}$ into an algebra F' if it preserves Boolean structure, i.e. if $(\neg A)' = \neg A'$, $(A \vee B)' = A' \vee B'$, and $\bot' = \bot$. The function $'$ such that $A' = \langle A, A \rangle$ is easily verified to be an embedding from $\mathcal{F}$ to the G-algebra $\mathcal{F}'$. If $'$ is an embedding and I an input, then I' is the input that results from I by replacing E_i in the originating partition by E_i', for all i. A probability function $\widehat{P}$ on $\mathcal{F}'$ is an *extension* of P on $\mathcal{F}$, relative to embedding $'$, if for all $A \in \mathcal{F}$, $P(A) = x$ implies $\widehat{P}(A') = x$. An expected value function $\widehat{V}$ is an extension of V, relative to embedding $'$, if for all $A \in \mathcal{F}$, $V(A) = x$ implies $\widehat{V}(A') = x$. If $\widehat{P}$ is an extension of P and $\widehat{V}$ is an extension of V relative to the same embedding, $\langle \widehat{P}, \widehat{V} \rangle$ is an extension of $\langle P, V \rangle$ relative to that embedding.

DBN Extension Lemma For every $\langle P, V \rangle$ on $\mathcal{F}$ such that $0 < V(T) < 1$, there is an algebra $\mathcal{F}'$, an embedding $'$, an extension $\langle \widehat{P}, \widehat{V} \rangle$ and a proposition $G \in \mathcal{F}'$ such that for all $A \in F$, $\widehat{P}(G|A') = \widehat{V}(A)$. Moreover, $\widehat{P}_{I'}(A) = \widehat{P_I}(A)$ for all $A \in \mathcal{F}'$.

Proof. Over the G-algebra of W, define $\widehat{P}(\langle A, B \rangle) = P(A)V(A) + P(B)(1 - V(B))$ for all $A, B \in \mathcal{F}$, and $\widehat{V}(A) = P(G|A)$ for all $A \in \mathcal{F}'$. The verification that $\widehat{P}$ is an extension of P and the other assertions of the Lemma are left to the reader.

It is not a limitation on the applicability of the DBN Extension Lemma that $V(T)$ needs to be between 0 and 1, since we can always rescale.

From the DBN Extension Lemma, we can conclude that any agent can be modelled by a pair $\langle \widehat{P}, \widehat{P}_G \rangle$ for which DBN holds, and continues to hold under updating.

5 From DBN to DAB

If an agent satisfies DBN, we could conclude that she satisfies DAB, given the Conditional Construal of Conditional Probability ("CCCP"), according to which for all A, there is a proposition C (heuristically, the conditional "if A then G") such that $P(C) = P(G|A)$ for all probability functions P. We could simply take C to be our A°. Unfortunately, the CCCP has been the target of various triviality results on its own.

However, we ought not to give up too quickly when trying to build a bridge from DBN to DAB. For there are also positive results about CCCP, albeit partial ones. I will make use of such a positive result due to van Fraassen [1976]. His idea is to construct, for a given algebra $\mathcal{F}$ a larger one in which there is a conditional $A \to B$ for all $A, B \in \mathcal{F}$, and to extend the probability function on $\mathcal{F}$ in such a way that CCCP holds in the larger algebra.

If $\mathcal{F}$ is an algebra of subsets of W, the *Bernoulli algebra* $\mathcal{F}'$ of W is an algebra of subsets of W^ω. An element of $\mathcal{F}'$ is a map from the natural numbers to W, or equivalently, a denumerable sequence $\langle w_1, w_2, ... \rangle$ of elements of W. If $A_i \in \mathcal{F}$, for $i = 1, ..., n$, then the notation $\langle A_1, ..., A_n \rangle$ will be used for the set of those $w \in W'$ such that $w_i \in A_i$ for $i = 1, ..., n$. The Bernoulli algebra is generated by sets of this form.

Van Fraassen then defines a conditional on the larger algebra. For our purposes, we can hold the consequent fixed and restrict ourselves to $A \to G$, or A°. For $A \in \mathcal{F}'$, define A^i to be $\langle \neg A, ... \neg A, A \wedge G \rangle$, where this sequence has length $i + 1$, and define $A^\circ = \bigvee_{i=0}^{\infty} A^i$. In other words, A° is the set of sequences in which the first element that is in A is also in G.

The next lemma is essentially the Theorem stated at the top of p. 295 of van Fraassen [1976].

CCCP Extension Lemma If P is a probability function on $\mathcal{F}$ and $G \in \mathcal{F}$, there exists an algebra $\mathcal{F}'$, an embedding $'$, an extension $\widehat{P}$ of P and a function $^\circ$ defined on $\{A \in \mathcal{F}' : \exists B \in \mathcal{F}$ such that $A = B'\}$ such that the following equation holds: $\widehat{P}(A'^\circ) = \widehat{P}(G|A)$, for all $A \in \mathcal{F}$.

Proof. Clearly, the mapping $'$ such that $A' = \langle A \rangle$ is an embedding from

$\mathcal{F}$ to the Bernoulli algebra $\mathcal{F}'$. The product measure defined on generating sets by $\widehat{P}(\langle A_1, ..., A_n\rangle) := P(A_1) \cdot ... \cdot P(A_n)$, and extended to other members of $\mathcal{F}'$ by additivity, is an extension of P (since $\widehat{P}(A') = P(A)$ for all $A \in \mathcal{F}$.

It remains to verify that the equation holds: $\widehat{P}(A'^{\circ}) = \widehat{P}(\bigvee_{i=0}^{\infty} A_i) = \sum_{i=0}^{\infty} \widehat{P}(A^i) = \sum_{i=0}^{\infty} P(\neg A)^i P(A \wedge G) = P(A \wedge G) \cdot \sum_{i=0}^{\infty} P(\neg A)^i = P(A \wedge G) \cdot \frac{1}{1-P(\neg A)} = P(A \wedge G) \cdot \frac{1}{P(A)} = \frac{P(A \wedge G)}{P(A} = \frac{\widehat{P}(A \wedge G)}{\widehat{P}(A)} = \widehat{P}(G|A)$.

The DBN Extension Lemma tells us that the order in which we apply the operation of extension to the G-algebra and the operation of Jeffrey conditionalization does not matter: that diagram commutes. In contrast, it matters whether we first extend to the Bernoulli algebra and then Jeffrey conditionalize, or the other way round: $\widehat{P}_{I'}(A) \neq \widehat{P_I}(A)$ for some $A \in \mathcal{F}'$. This is to be expected, given Lewis's result that any equation of $V(A)$ and $P(A^{\circ}$ would be shorn apart by Jeffrey conditionalization. I thus propose that a defender of DAB replaces Jeffrey conditionalization with something else. The *contracted Jeffrey conditionalization* of $\widehat{P}$ on I' is simply $\widehat{P_I}$. We contract from the extension to the initial algebra, deploy standard Jeffrey conditionalization, and extend again.

This move may appear unduly *ad hoc*. However, there are good independent reasons to deny that Jeffrey conditionalization belongs to the non-negotiable core of decision theory. The debate about the best epistemic policy in a Sleeping Beauty situation suggests that when credence functions are defined over centered rather than uncentered worlds, conditionalization or Jeffrey conditionalization may not be the best policy. In fact, Christopher Meacham adduces strong arguments in favour of what he calls "compartmentalized conditionalization," which rests on the same idea as contracted Jeffrey conditionalization.

An embedding in a G-algebra allows us to represent an agent as satisfying DBN. An embedding in a Bernoulli algebra allows us to represent an agent satisfying DBN as satisfying DAB as well. We are now in a position to put the two constructions together, and show that every agent can be represented as satisfying DAB.

DAB Extension Theorem For every $\langle P, V\rangle$ on $\mathcal{F}$, there is a Bernoulli algebra $\mathcal{F}'$ on $W \times W$, an embedding $'$, an extension $\langle \widehat{P}, \widehat{V} \rangle$ of $\langle P, V \rangle$, and a function $^\circ$ definded on $\{A \in \mathcal{F}' : \exists B \in \mathcal{F}$ such that $A = B'\}$ such that for all $A \in F$, $\widehat{V}(A') = \widehat{P}(A'^\circ)$ holds, and continues to hold under contracted conditionalization.

Proof. Left to the reader, using the previous Lemmas.

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